MEDICAL directors use several inter-related mortality measures to express risk. Two of them — the mortality ratio (MR) and the excess death rate (EDR) — are used quite extensively, partly because they each bear a simple relationship to "expected" mortality rates, and partly because they relate directly to the day-to-day business of arriving at table ratings and flat extras. A third measure of mortality risk — life expectancy — intrudes into our thinking when we are called upon to estimate years of life remaining, such as in structured settlements or second-survivor life. As often as not, the life in question is an impaired or substandard life. It is then that the medical director is confronted with the non-linear relationship between attained age, degree of substandardness (effective mortality rates), and ultimate survival (life expectancy).

The MR and EDR are fairly straightforward measures of incremental mortality risk, the former proportionate (a ratio), the latter by simple difference. Life expectancy, however, has no simple formula, and no intuitive feel to it. As new medical directors we quickly discovered that doubling the mortality rate does not halve the life expectancy. The survival function through which age and mortality-risk operate is a complex equation in which the "force of mortality" (px) is part of an exponent of e. This µx is not the same as qx (the annual mortality rate at age x) but is more like an instantaneous mortality rate, with values >1 therefore possible. In some survival models Jax has the general form A+Bc sub x in which the "force of mortality" is itself broken down into component forces. Very crudely speaking, the A,B,C's of mortality are the weights given to Accident, Behavioral hazards (e.g., smoking, acquired diseases, and controllable or correctable risk factors), and cellular aging operating at any age "x."

What Singer does in his article "A Method of Relating Life Expectancy in the U.S. Population Life Table to Excess Mortality" (pages 32-41), is to argue for "EDR vs. age" as a more useful method of arriving at "years of life remaining" than a table of "MR vs. age." He cites three primary reasons why the EDR approach might be preferred: 1) as an arithmetical increment and not a proportionate one, EDR is less susceptible to significant distortion at higher ages, where the denominator of the MR is changing and ever-increasing; 2) this "stability" makes it more useful for survival analysis over extended time-frames (e.g., 10-20 years or more); and 3) when one is projecting future survival from short-term (5 and 10 year) clinical or insurance studies, it is often easier and more valid to estimate the EDR-tail that would follow the measured survival curve, than to estimate a meaningful or durable subsequent MR. In other words, if one has to assign a single excess-mortality value to a span of years, and if that span of years is relatively long or involves persons of higher ages, then the EDR may be more useful and dependable than an annualized MR.

Singer, in his inimitable way, does much more than this, of course. He takes us inside a life-table (1979-81 U.S.) and reveals some of its inner workings. At last year's Advanced Mortality Methodology workshop, he pulled out this very table and asked if anyone knew what column 6 (Tx) was, or what it was used for. Never having need of these tables for much beyond qx, we all artfully demurred. With hints from Dr. Singer, we soon deduced that they were the sum of column 5 (Y. L×) from that age x to the end of the table, and that they were among the few entries on the table that were figured from the "bottom up," as it were, instead of from the top down. So each Lx represented the total person-years of life remaining to those individuals currently age x or higher. With Dr. Singer's Socratic prodding, we deduced that column 5 itself was approximated by subtracting 1/2 dX from Lx (i.e., 1/2 of column 4 from the entry in column 3 at age x) and that the life expectancy (column 7, Ex) was Tx/Lx.

The life-table derivation of Ex requires assumption of a "stationary population," a demographic concept relying
on a "constant density of births" such that the survival function doesn't depend on time of birth but only on birth-rate (b). Accordingly, b becomes the radix or cohort-number of the table (ε), and columns 3 and 4 are labelled "of 100,000 born alive." While the tabulation in the 1979-81 U.S. Table chiefly reflects factors and forces operant at that census, the lives upon which those factors act are a mixed pool of lives. The 50-year-old white males who contribute 89,787 person-years to Tx and hence to the life-expectancy calculation are those who survived their first year of life subject to the qx for 1930; their 10th year of life subject to the q10 for 1940; and so on. So the grand life-table of 1979-81 U.S. has both contemporaneous and longitudinal aspects to its cross-sectional snapshot of survival and mortality.

While the tabular life-expectancy (Tx/ε) is not a "median life-expectancy" or the life span expected of half of those still alive (e.g., the half-life of the cohort), it closely approximates this number. For example, the tabular life expectancy of a 9 year-old, white male (US 1979-81) is 62.96 years, but the number alive at start of that interval (98,381) does not become reduced by 1/2 (49,191) until around age 74 (i.e., 65 years later). For higher ages, the difference between tabular and median life-expectancy becomes smaller (e.g., for 50 year-old white male, tabular life-expectancy is 25.26 years, and median survival is to age 70 or 75, or 25-26 years). Another phenomenon worth noting is that projected age of eventual-death (e.g., attained age plus life expectancy) creeps upward as attained age increases. The age at death projected for a 9 year-old (U.S. 1979-81) is 71.96 years, but for a 50 year-old becomes 75.26 years. As life's hazards are passed and attained age increases, the survivors become a select group from whom even longer survival can be expected.

Singer's Table 5 provides a useful and illustrative tabulation of life expectancy for white males, U.S. population 1979-81, shown as age vs. progressively higher EDR. Somewhere around EDR of 100-200, the EDR so dwarfs the baseline qx that for all practical purposes, life expectancy is "flat" (i.e., qx virtually equals EDR, for all intents and purposes). A table for insured lives, rather than U.S. population, can be created using the spreadsheet approach Singer describes. One such table, based on 1980 IoA Assured Male (5-year) Ultimate tables, will be available in the new 3rd edition of Brackenridge, due out later this year. It will appear opposite an updated "age vs. MR" table of life expectancy, permitting the medical director to choose an EDR or MR approach (and compare results).

Declining Exponential Approximation to Life Expectancy (DEALE)

The DEALE of Beck, Pauker, and Kassirer2 has some usefulness over short spans of time (spans over which a declining exponential curve approximates the relevant portion of a Gompertz-shaped survival curve) or in populations (like many cancers) where the survival curves are roughly declining-exponential in configuration. And they are also useful for the decision-analysis purposes they are intended to support (combining several highly-substandard co-morbid conditions to facilitate choice from among several possible therapeutic regimens). Life expectancy solutions via DEALE are reputed to be "within 12%" of expectancies calculated by other means, (p. 887) and what they may lack in exactness, is more than made up for in ease and speed of calculation. One thing to be aware of is that their notation uses μ (mu) for population (e.g., μasr = age-sex-race specific mu) and impairments (e.g., μZ = impairment-specific extra mortality/1000 attributable to condition "Z") where our standard notation would use qx and impairment-specific EDR's. This μ in DEALE parlance does not mean "force of mortality" but rather an annualized mortality rate. Accordingly, when they take the life expectancy of a 40 year-old white male as 34 years, they assign a μ (age-specific annual mortality rate) of .029. Assuming 34 years is the median life-expectancy (t1/2), and assuming survival and mortality rates to be related as a declining exponential, then "μ" would be the inverse of 34 or 0.029. However, tabular qx for U.S. Males (1979-81) is .003, an order of magnitude different. As a practical matter, this does not demolish DEALE estimates, since the EDR's are generally far higher than the μ's or qx's, and since the DEALE calculations are largely self-correcting, with μasr added in and then subtracted out again at various stages. Beyond these limitations, DEALE has some features to commend it for curbside estimations of life expectancy.

From Expectancy to Expected

Also appearing in this issue are two abstracts: the promised second part of SHEP (Systolic Hypertension in Elderly Persons) data-analysis, (pages 28-31) and some NASCET (North American Symptomatic Carotid Endarterectomy Trial) morbidity and mortality results (pages 42-46). Together with the two abstracts of last issue (part I of SHEP and a study of endarterectomy in Asymptomatic Carotid Bruit), these abstracts form an interesting cluster. They also amply demonstrate the kinds of resourcefulness medical directors are challenged to summon in developing suitable "expected
"rates" for comparative morbidity and mortality analysis.

Take the NASCET abstract, for example. In the absence of detailed age-sex composition data, or even a mean-age to use with the overall M/F ratio, assumptions had to be made that mean-age was near the median-age, and that "mean q" was approximated by "q" for the mean age plus 3 years. Reliance on these assumptions gives the Surgical group a mortality experience (Table 3) not very different from Group Life or U.S. population expected. Reasons for this could be several. One possibility is that the effects of selection (i.e., exclusion criteria) and treatment (medical and surgical therapeutic effects) did, in fact, result in this kind of favorable mortality experience. But it is also possible that the mean age does not equal the median (i.e., that the age distribution is not a "normal distribution") but might be less than that since the age range was so broad (31-79 years). Or perhaps the sex-distribution - if known in detail - would weight the "mean q" more heavily toward lesser expected mortality for the group (perhaps at or below "q" for the mean age"). Astute readers can probably think of yet other possibilities. If NASCET does release additional information, it will be a worthwhile exercise to do a more exact calculation of expected mean q from detailed compositional information.

The derivation of suitable "expected" rates is among the more tedious, but critical, parts of developing a mortality abstract, and often is more time-consuming than extracting interval and average q's from "observed" data available as cumulative P or Q information. Even more problematic is deriving "expected" rates for morbidity abstracts, but here, Singer has shown the way and broken the ground for others to follow. His two SHEP abstracts and an earlier abstract on recurrent MI risk show how to make more and better use of Framingham and similar studies. While the estimation of $r'$ used for the NASCET stroke data would hardly satisfy the purist in any of us, it does provide a benchmark for comparison that is at least as good as the assumptions which support it - and is at least a forgivable shortcut, given the absence of detailed information with which to proceed any differently. While one can quibble about whether a better $r'$ would be 7.8 or even 15/1000, the point hopefully gets across that approximations and "shortcuts" can permit the medical director to make reasonable ballpark estimates. And for some purposes, that is a more useful starting point in evaluating severity of risk than either a WAG ("wishful or arbitrary guess" -- and other translations) or an "I don't know." Eventually what is needed is a library of values of $r'$ for different impairments, at different ages, and across different populations. Perhaps if morbidity and mortality abstraction becomes a larger-scale effort on the part of our profession, such a library will be built through these pages?

Advanced Mortality Methodology

A "P's and Q's" commentary could not be "expected" to close without some pitch for abstract production. You will note in this issue of the Journal a course description and application form for the next Advanced Mortality Methodology workshop (page 89), which will be held on the Wed-Fri following this fall's Academy meeting in Boston. Dr. Singer has once again put together an enrichment program that can be of operational value as well as intellectual interest to the medical director. In the last issue of JIM you saw two abstracts that resulted from the 1991 workshop (Braun's "Life Table Analysis of AIDS Mortality" and Kita's "Mortality in Patients with Asymptomatic Carotid Bruit"). In the next issue, you will see MacKenzie's abstract of valve-replacement surgery long-term follow-up, which is also a product of the 1991 workshop. Dr. Singer helps participants choose an article for the workshop that is suitable for abstraction and which contains valuable data of publishable merit. And the various approaches to handling expected morbidity and mortality are covered.

References